HEEDS Technical Tip – Searching the Log Space of a Design Variable

Level: Intermediate

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Introduction

When continuous design variables are defined in HEEDS Professional they are not treated as truly continuous. They are actually discretized into variables with many small, equally spaced values. The size of these increments is decided by the *resolution* assigned to the variables. By default, this value is set to 101. This value means that the design variables are discretized into 101 values, with consecutive values spaced equally from each other.

This is done because in most engineering applications, it is not necessary to design to an exceedingly high precision. In this way, HEEDS simplifies the search space, and does not miss major design improvements while trying to improve the design by tiny fractions of a percent.

One side effect of this discretization is that when the design variable range spans many orders of magnitude, this search of equally spaced values in the *linear space* may yield a poorly posed optimization statement. Design variables with these very large ranges do not commonly appear in design of the size and shape of physical components, but they do appear in situations such as curve fitting, chemical rate constants, etc. Another common example is in calibrating nonlinear material models. Reasonable ranges for the design variables (the material model parameters) are not always obvious, so large ranges are often used, allowing variables to range from very small (near-zero, but definitely not zero) to very large. Take, for example, the situation in which the design variable ranges from a minimum of 1 to a maximum of 1,000,001 (total size of the range is 1,000,000). For a resolution of 101, there are 101 equally spaced values, spaced in increments of 10,000. That means that the next smallest value after the minimum is 10,001.

An alternative is to search the *log space*, which is to say that the values are evenly spaced when plotted on a log-scale. In this case, with a resolution of 101, the next smallest value after the minimum is 1.1482. (Clearly, with a larger resolution, this spacing can be made as small as desired.) This is illustrated in Figure 1 and Figure 2.

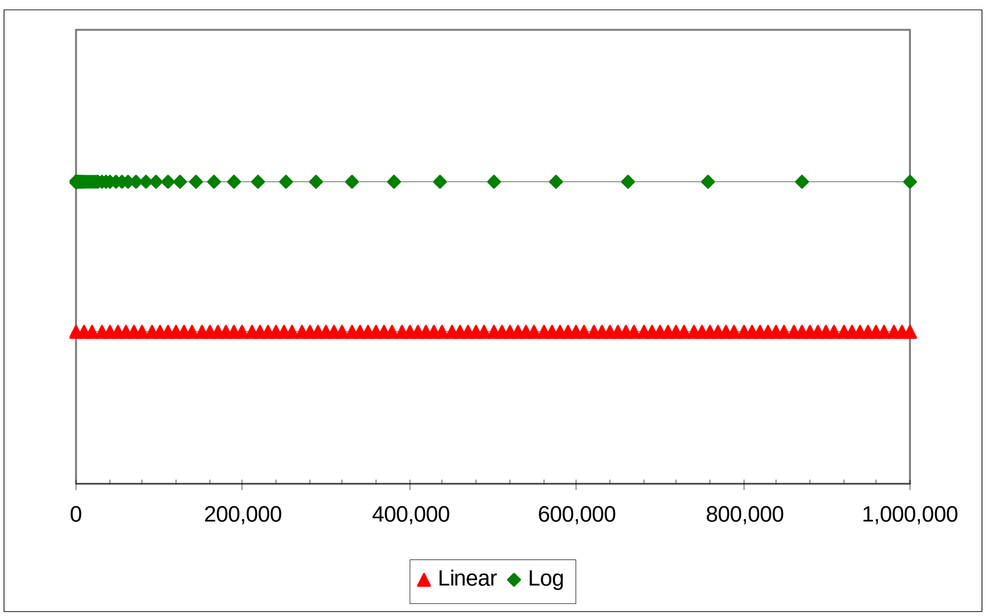


Figure 1. Possible values for a design variable with resolution of 101, minimum 1, and maximum 1,000,001. The red markers indicate the possible values if a search of the linear space is performed. The green markers indicate the possible values if a search of the log space is performed.

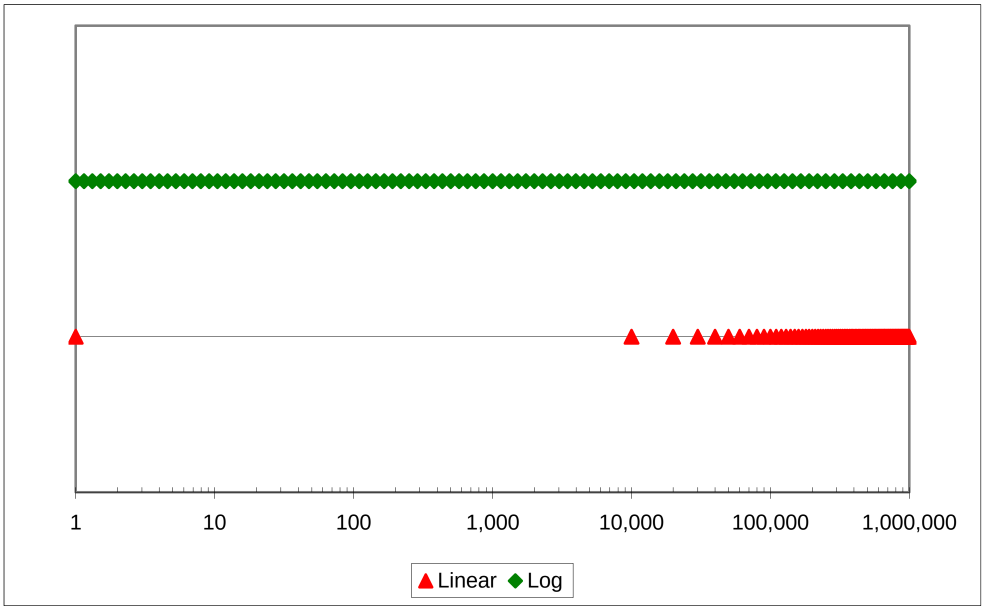


Figure 2. Same data as shown in Figure 1, except plotted on a log scale.

The benefit of searching the log space instead of the linear space is that the designer can now approximate the value of the design variable within a certain ***percentage*** of its value instead of specifying an ***absolute*** precision. In this example, for the designer to be able to design to within 1% of the actual value of the design variable, the variable would need a resolution of about 100 million if searching the linear space, but only about 1,400 if searching the log space. The latter problem is posed better, and allows a more efficient search.

HEEDS Setup

To search the log space for a variable in HEEDS, simply create two variables in HEEDS for each actual design variable to be log-searched. The first will be the actual value of the design variable, *DVar1*. The second will be the variable that is actually varied directly by HEEDS, which is named, in our example, *order_DVar1*. Set *DVar1* to be a dependent variable with the formula "10^*order_DVar1*". The variable *order_DVar1* will be continuous and vary from 0 to 6, shown in Figure 3. This will allow DVar1 to vary from 1 (10^0) to 1,000,000 (10^6), but have equally spaced intervals in the log space. The resolution applied to the variable *order_DVar1* will decide to which percentage precision the variable *DVar1* will be designed.

Project Variables

	Name	Type	Min	Baseline	Max	Discrete Set	Formula
1	DVar1	Dependent					10^order_DVar1
2	order_DVar1	Continuous	0	1	6		

Figure 3. Design variable definition for log space search of DVar1.

It is important to note that when tagging, only the *DVar1* variable will be tagged in input files. The second variable, *order_DVar1*, is not tagged since it is only used to define the value of *DVar1*. To do this for more than one design variable, simply repeat the steps so there is a *DvarN* and *order_DvarN* variable for each design variable to be treated logarithmically. It is perfectly acceptable to mix some design variables in the linear space with others in the log space. Note that if the entire range of a variable is negative, a log search may be done by defining *DVar1* to be "-(10^*order_DVar1*)", and defining the limits of *order_DVar1* as if they applied to the absolute value of *DVar1*.

Calculating the Resolution to Use for Log Space Search

Most users, for most purposes, will simply use the default resolution of the *order_DVarN* variable to achieve enough resolution in their answers. Often 101 steps are enough, but for some applications perhaps a thousand or ten thousand steps are more appropriate. **If that is adequate for your purposes, and if all of your variables' values are positive, you do not need to read further.** However, if desired, calculations may be done to find the minimum number of steps that assures that the maximum percentage change between steps is less than some desired threshold. Log transformation may also be used even if a variable has some negative values, as described below.

Appropriate search in the log space may be done regardless of the overall range of values or step sizes (percentage differences) required. An example can help to clarify the procedure: a design variable range of 0.00002 – 300 can be fitted to step changes no larger than 0.1 percent by using a resolution of 16,532 in HEEDS. That is calculated as follows:

1. Calculate the second-lowest value desired: in this case, 0.00002002 (for a 0.1 percent change between values)
2. Calculate the difference of the logarithms of the two lowest values: $\log_{10}(0.00002002) - \log_{10}(0.00002) =$ 0.00043408
3. Calculate how many steps of that size are needed between the $\log_{10}(\ *\ )$ of the minimum and the maximum values of the range, i.e., in this case:

   $[\log_{10}(300) - \log_{10}(0.00002)]$ / 0.00043408 =

   [2.47712125 – (-4.69897)] / 0.00043508 = 7.17609126 / 0.00043409 = **16531.8212**

   (a HEEDS resolution of 16,532 steps or larger is sufficient to space successive steps of the design variable within 0.1% of each other)

If Negative Values Are Needed in the Range of *DVarN*:

So long as both the maximum and minimum values in the range are both positive (or both negative, in which case the dependent variable may simply have a minus sign inserted in the formula), the procedure outlined above is adequate.

It is not as common in engineering problems, but does occur, where it is not known whether a variable has a positive or negative value. That case may still make use of a log transform, if done with care. The value of $\log_{10}(0)$ is undefined – the logarithm approaches negative infinity as the argument approaches 0. Of course, logarithms of negative numbers are not defined, either. However, that does not mean that the user may not use the logarithmic transformation idea. The approach will be to first SHIFT the data so that all values are positive (or, in fact, 1 or greater, for simplicity) before taking the logarithm, and then defining *order_DVarN,* the log of the shifted data, to range from 0.0 upward to the maximum shifted value, and then "undoing" the shift when defining *DVarN* from *order_DVarN*.

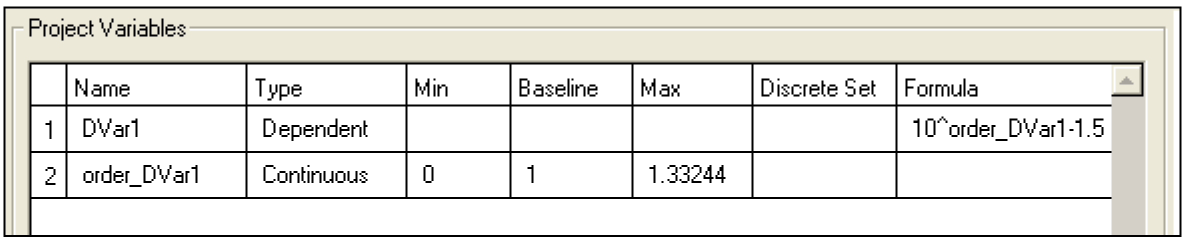
Let's look at an example of this. Assume that the desired range of values is from -0.5 to +20, that we want steps not larger than 0.1% of the adjacent value throughout the range, and that near 0, we want steps no larger than 0.005. In our minds, we will shift the values so that they range from +1.0 to +21.5, adding 1.5 to each value in the linear range. Then we will define *order_DVar1* as the $\log_{10}(\ *\ )$ of these shifted values, so it will range from $\log_{10}(1) = 0$ to $\log_{10}(21.5) =$ 1.3324385. Now the question is, what step size (or how many steps) is needed between these two values? There are two places in the range of *order_DVar1* that we must consider:

1. At the maximum of *order_DVar1* = $\log_{10}(21.5)$ = 1.3324385, which will ultimately transform to +20 in the linear space (this will also cover the minimum part of the range)
2. At *order_DVar1* = $\log_{10}(1.5)$, which will ultimately transform to 0.0 in the linear space. This will treat the portion of the range near 0.0, where a percentage specification (like 0.1% of adjacent value) is meaningless (0.1% of 0 would give a step size of 0, so we need some other specification of the minimum step size in this vicinity).

We want to choose a step size so that the percentage changes of the steps in BOTH of these regions is acceptably small.

1. At $\log_{10}(21.5)$ = 1.3324385, the previous step should be at least $\log_{10}(21.5 * .999)$, or $\log_{10}(21.4785)$ = 1.33200395. The difference in logs is +0.00043451. The number of steps needed, then, to get step sizes no larger than the desired 0.1% change is range / step_size = 1.3324385 / 0.00043451 = 3066.531, or at least 3067 steps.
2. Check the step size when *DVar1* is near 0 (so *order_DVar1* would be near $\log_{10}(1.5)$. Now $\log_{10}(1.5)$ is 0.1760913, and the desired maximum step size here is 0.005, so we calculate $\log_{10}(1.5005)$, which is 0.176236. So the step size wanted here, in log space, is 0.176236 - 0.1760913 = 0.0001447. To achieve this, we would need a resolution of 1.3324385 / 0.0001447 = 9,208.2827, or at least 9,209 steps. Note that this is many more steps than the 3067 steps needed to get a maximum change of 0.1% elsewhere in the range.

Given those two calculations, we can see that we need a resolution of at least 9209 steps to satisfy both conditions. This example is setup in HEEDS in Figure 4. The user simply sets the formula for *DVar1* to be equal to “10^*order_DVar1* – 1.5”. This results in the range of -0.5 to +20 for DVar1, with equally spaced increments in the log space. Not shown is that the user would also need to specify a resolution of 9209 for *order_DVar1* in the **Assembly** tab to get the desired steps size.


Project Variables

	Name	Type	Min	Baseline	Max	Discrete Set	Formula
1	DVar1	Dependent					10^order_DVar1-1.5
2	order_DVar1	Continuous	0	1	1.33244		

Figure 4. Design variable definition for log space search of DVar1 where zero is included in the desired range.

Conclusion

In order to properly search design spaces where the design variable spans many orders of magnitude, it is important to adjust the search to use the log space instead of the linear space. This is a more robust optimization statement, and better encourages efficient search. To search the log space in HEEDS, the user only has to add a simple formula and calculate the range of the *order_DvarN* variable.