

HEEDS Technical Tip - Imposing Constraints on Design Variables

Level: Intermediate

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Introduction

When solving engineering design optimization problems, it is common to have design variables that are semi-independent. That is, the variables vary independently within a range, but the range over which they vary is dependent on the values taken by other variables. For example, three variables may be independent, but the sum of these three variables must be equal to a prescribed value.

In general, there are two ways to deal with these types of problems:

1. **Impose a constraint on the design variable values via a formula-based response.**
While this approach is the simplest in terms of problem set up, it has the disadvantage of allowing the generation of many infeasible designs, which makes the design space more difficult to search. Further, since the constraint might not be evaluated until after the analysis is complete, a significant amount of CPU time will be wasted evaluating infeasible designs.
2. **Redefine the design variables such that HEEDS only creates designs that meet the imposed constraints.**
This requires some planning to establish formulas for the ranges and may require additional project variables, but the number of independent design variables is the same or possibly even reduced.

This article will describe method 2 in detail for three example problems.

Example 1 – Composite Plate

A composite plate is to be designed by varying the thickness of the individual plies. The plate has 8 plies, and the total thickness of the plate must be equal to 3 mm. If $thick_n$ is the thickness of the n^{th} ply, then

$$\sum_{n=1}^8 thick_n = 3 \text{ mm}.$$

The strategy used to set up this problem is described in the steps below, and the result is shown in Figure 1.

1. Define a parameter, $Total_Thickness$, and set its value to 3 (mm).
2. Define eight continuous variables, $p1$ through $p8$, each with upper bounds less than 1. p_k ($k = 1$ to 8) represents the fraction of the remaining thickness that will be assigned to the k -th ply after plies 1 through $k-1$ have already been assigned a thickness.
3. Define $thick1$ as a dependent variable equal to $p1*Total_Thickness$. Thus, $thick1$ can only take on values smaller than $Total_Thickness$.
4. Define $thick2$ as a dependent variable equal to $p2*(Total_Thickness-thick1)$. Thus, $thick2$ can only take on values smaller than the remaining thickness after $thick1$ has been assigned.
5. Define $thick3$ as a dependent variable equal to $p3*(Total_Thickness-thick1-thick2)$. Thus, $thick3$ can only take on values smaller than the remaining thickness after $thick1$ and $thick2$ have been assigned.
6. Repeat for $thick4$ through $thick7$, subtracting the previous thickness each time
7. Define $thick8$ as a dependent variable equal to $Total_Thickness-thick1-thick2-thick3-thick4-thick5-thick6-thick7$, which is what is left of the 3 mm thickness after assigning the other 7 thicknesses.
8. Assign upper and lower bounds on $p1$ through $p7$. These are somewhat arbitrary, though the upper bounds should never be equal to 1, because this will result in plies with 0 thickness.

As shown in Figure 1, this problem definition requires additional project variables, but the actual number of design variables has been reduced from 8 to 7 ($p1$ through $p7$). This guarantees that every design will have a plate thickness of exactly 3 mm and reduces the size of the design space.

Project Variables							
	Name	Type	Min	Baseline	Max	Discrete Set	Formula
1	Total_Thickness	Parameter		3			
2	thick1	Dependent					$p1 * \text{Total_Thickness}$
3	thick2	Dependent					$p2 * (\text{Total_Thickness} - \text{thick1})$
4	thick3	Dependent					$p3 * (\text{Total_Thickness} - \text{thick1} - \text{thick2})$
5	thick4	Dependent					$p4 * (\text{Total_Thickness} - \text{thick1} - \text{thick2} - \text{thick3})$
6	thick5	Dependent					$p5 * (\text{Total_Thickness} - \text{thick1} - \text{thick2} - \text{thick3} - \text{thick4})$
7	thick6	Dependent					$p6 * (\text{Total_Thickness} - \text{thick1} - \text{thick2} - \text{thick3} - \text{thick4} - \text{thick5})$
8	thick7	Dependent					$p7 * (\text{Total_Thickness} - \text{thick1} - \text{thick2} - \text{thick3} - \text{thick4} - \text{thick5} - \text{thick6})$
9	thick8	Dependent					$\text{Total_Thickness} - \text{thick1} - \text{thick2} - \text{thick3} - \text{thick4} - \text{thick5} - \text{thick6} - \text{thick7}$
10	p1	Continuous	0.05	0.125	0.3		
11	p2	Continuous	0.05	0.142857	0.4		
12	p3	Continuous	0.1	0.166667	0.5		
13	p4	Continuous	0.15	0.2	0.6		
14	p5	Continuous	0.2	0.25	0.7		
15	p6	Continuous	0.25	0.333333	0.8		
16	p7	Continuous	0.3	0.5	0.9		

Figure 1. Project Variables for Example 1.

Example 2 – Material Model

A material model with 3 parameters (C_1 , C_2 , and C_3) is to be calibrated to experimental data. The model is only valid when $C_1 \geq C_2 \geq C_3 \geq 0$.

The strategy employed to set up this problem is described in the steps below, and the result is shown in Figure 2.

1. Define $C1$ as a continuous variable.
2. Define $p2$ and $p3$ as continuous variables with a range of (0,1).
3. Define $C2$ as a dependent variable equal to $p2 * C1$.
4. Define $C3$ as a dependent variable equal to $p3 * C2$.

Again, this problem statement requires more project variables, but in this case the total number of independent design variables stays the same. This is due to the inequality constraint, while in Example #1 an equality constraint was imposed, resulting in one less design variable.

Project Variables							
	Name	Type	Min	Baseline	Max	Discrete Set	Formula
1	C1	Continuous	0	4	5		
2	C2	Dependent					$p2 * C1$
3	C3	Dependent					$p3 * C2$
4	p2	Continuous	0	0.6	1		
5	p3	Continuous	0	0.25	1		

Figure 2. Project Variables for Example 2.

Example 3 – Tailor Welded Blank

A tailor welded blank (TWB) consists of two pieces of sheet metal that are welded together prior to stamping. The thicknesses of the parts are allowed to vary independently, but in order to achieve a proper weld the ratio of the two thicknesses must satisfy certain limits. In this example consider the case in which the thickness ratio must be greater than or equal to 0.5 and less than or equal to 2 (i.e., the larger part can be no more than twice as thick as the smaller part). If the

thickness of the parts are t_1 and t_2 , then $0.5 \leq \frac{t_1}{t_2} \leq 2$.

The strategy employed to set up this problem is described in the steps below, and the result is shown in Figure 3.

1. Define t_1 as a continuous variable.
2. Define $t1_over_t2$ as a continuous variable that can range from 0.5 to 2 (the bounds to be imposed on the ratio).
3. Define t_2 as a dependent variable that is equal to $(1/t1_over_t2)*t_1$.

Project Variables							
	Name	Type	Min	Baseline	Max	Discrete Set	Formula
1	t1	Continuous	0.1	0.5	0.875		
2	t2	Dependent					$(1/t1_over_t2)*t1$
3	t1_over_t2	Continuous	0.5	1	2		

Figure 3. Project Variables for Example 3.

Conclusion

To efficiently accommodate semi-independent design variables, a method of redefining the optimization problem was demonstrated. This technique has the benefit of generating only feasible designs (with respect to the constraint on design variables), and in some cases has the added benefit of removing an independent design variable from the design space.