Nonlinear Material Characterization using HEEDS and ABAQUS

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Introduction

With increasing applications of modern materials in their highly nonlinear range of behavior, the need for improved material models to predict this nonlinear behavior continues to be an important problem. Both mechanistic and phenomenological material models contain parameters that must be determined by best matching experimental data over a range of loading conditions. Assuming the underlying model is appropriate for the material under consideration, the determination of the unknown parameters in the model can be represented as a parameter optimization problem. In this study, a new hybrid optimization strategy is discussed and applied to two material characterization problems: rate-sensitive hyper-elastic polymers and rubbers used in automotive engine mount applications.

Modeling Nonlinear Materials

Linear elastic materials can be characterized by experimentally determining only two coefficients, E and υ. Hyperelastic materials, such as those examined here, require additional material coefficients to represent the nonlinear and/or strain rate behavior. Once a material model is chosen, the coefficients of the model must be determined by performing a best fit to experimental data. This can be cast as a parameter optimization problem, where the design variables are the coefficients and the objective is to minimize the difference between the predicted load curves and those obtained from experimental testing.

In the examples below, HEEDS and its search algorithm SHERPA were used for optimization, and ABAQUS was used for the nonlinear analysis. HEEDS is a commercial design optimization software package [1]. Its proprietary search algorithm SHERPA is a hybrid adaptive approach that employs several global and local search methods and strategies simultaneously to identify an optimized solution. SHERPA also adaptively refines its strategies during the search as it learns about the design space. ABAQUS is a commercial software package for nonlinear finite element analysis [2]. The procedure used for the following examples is illustrated by the flow chart in Figure 1.

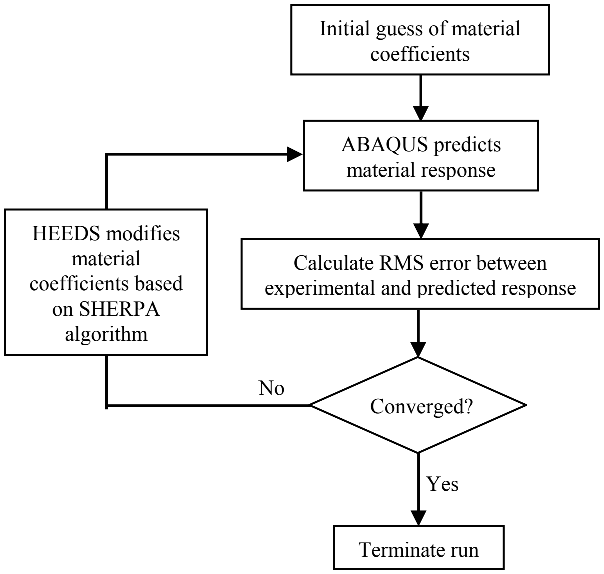

Figure 1. Flow chart of the general procedure for material characterization using parametric design optimization.

Rubber Engine Mounts

An existing engine mount used for NVH applications in automobiles was to be analyzed. The geometry was known, and the secant stiffness for loading in three directions had been found through experiments. Material data, however, was not available. In this study, the material model was obtained through an inverse approach, wherein the material constants of the material model were obtained from a best fit of the structural response of the mount.

The Mooney-Rivlin model was chosen for this rubber. This model is a special case of the polynomial family of hyperelastic models, and the strain energy potential function is given in equation (1).

$$U = C_{10}\left(\bar{I}_1 - 3\right) + C_{01}\left(\bar{I}_2 - 3\right) + \frac{1}{D_1}\left(J^{el} - 1\right)^2 \qquad (1)$$

where $\bar{I}_i$ are the deviatoric strain invariants, J^{el} is the elastic volume ratio, C_{10}, C_{01}, and D_1 are the coefficients of the potential function [3].

The coefficients C_{10} and C_{01} were the design variables, while D_1 was held constant. The objective was to minimize the root mean square (RMS) of the error between the predicted secant stiffness in each direction and the secant stiffness found from experiments:

$$RMS = \sqrt{\left(k_1 - k_{1,tgt}\right)^2 + \left(k_2 - k_{2,tgt}\right)^2 + \left(k_3 - k_{3,tgt}\right)^2} \tag{2}$$

The results of this study are summarized in Table 1. Shown in Figure 2 is a finite element mesh of the engine mount, with the 1, 2, and 3 directions shown. The bottom face of the mount is fixed and the loading is applied on the top face in the directions indicated.

Table 1. Results of material coefficient determination

Label	Target	Optimization	% Error
1-direction	65 N/mm	57.5 N/mm	11.5%
2- direction	407 N/mm	407.7 N/mm	1%
3- direction	43 N/mm	49 N/mm	13.9%

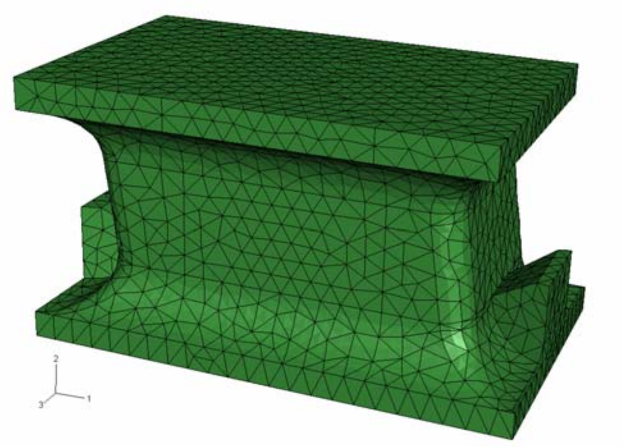

Figure 2. Rubber engine mount.

Rate-sensitive Polymer

A hyperelastic and viscoelastic model of a polymer was desired for use in a structural model. Load curves from experiments were available for several strain rates. The Neo-Hookean model was chosen to model the hyperelasticity of the polymer, and the rate-dependence was modeled with a four-term Prony series expansion of the relaxation modulus. The Neo-Hookean strain energy potential function is a special case of the polynomial family, and is given in equation (3). The Prony series is given in equation (4).

$$U = C_{10}\left(\bar{I}_1 - 3\right) + \frac{1}{D_1}\left(J^{el} - 1\right)^2 \tag{3}$$

where $\bar{I}_1$ is the first deviatoric strain invariant, J^{el} is the elastic volume ratio, C_{10} and D_1 are the coefficients of the potential function [3].

$$g_r(t) = 1 - \sum_{i=1}^{N} \bar{g}_i^P \left(1 - e^{\frac{-t}{\tau_i^G}}\right) \tag{4}$$

where N, $\bar{g}_i^P$, and τ_i^G are material constants [4].

For this problem, N was set equal to four. Since D_1 was held constant, this left five coefficients to act as design variables: C_{10} and the four $\bar{g}_i^P$ terms.

The RMS value of the predicted load curves and the curves found from experiment for several strain rates was minimized. Shown in Figure 3 are the predicted and experimental load curves found after running the optimization with HEEDS.

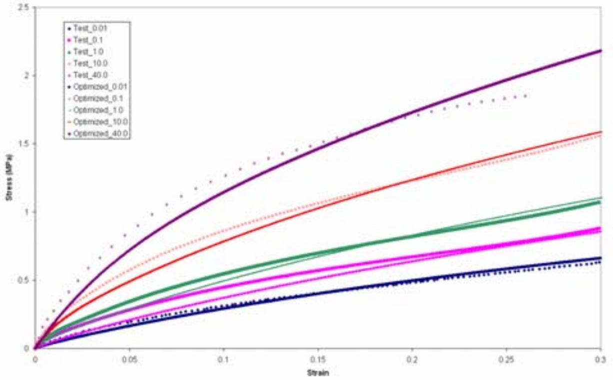

Figure 3. Comparison load curves from test data and optimization with HEEDS.

Especially at the lower strain rates, the results of optimization were far more accurate than were found manually.

Conclusions

In the preceding examples, design optimization was used to characterize a rubber engine mount and a rate-sensitive polymer. Design optimization using SHERPA was found to be an effective method for determining material coefficients.

References

[1] "The SHERPA Method", *HEEDS v5.1 User's Manual*, Red Cedar Technology, Inc., 2007, Chapter 10.

[2] *ABAQUS v6.6 Analysis User's Manual*, ABAQUS, Inc., 2006

[3] "Hyperelastic behavior of rubberlike materials", *ABAQUS v6.6 Analysis User's Manual*, ABAQUS, Inc., 2006, Section 17.5.1.

[4] "Time domain viscoelasticity", *ABAQUS v6.6 Analysis Manual*, ABAQUS, Inc., 2006, Section 17.7.1.